N69 01056

BELLCOMM, INC.
955 L'ENFANT PLAZA NORTH, S.W. WASHINGTON, D.C. 20024

SUBJECT: Targeting of the LM Descent Visibility Phase and "Extended" Braking Phase - Case 340

DATE: January 24, 1969

FROM: I. Silberstein

ABSTRACT

Recent changes in the LM descent trajectory and expected future changes point to the need of a rational procedure in the targeting of such trajectories.

This work outlines a method for the design of a visibility phase which conforms with a set of constraints similar to those used in the Apollo trajectory.

In addition, a method for extending the targets of a braking phase for an additional time T is presented. Together with the work of G. L. Bush and O. R. Pardo[1] on automatic targeting of the braking phase, this completes the solution of the targeting of a whole descent trajectory.

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MEMORANDUM FOR FILE

1.0 INTRODUCTION

Recent problems with the LM descent trajectory led to the consideration of a single phase landing, that is, the deletion of high gate. However, many of those involved in the problem of LM landing are not convinced that a single phase landing is the best solution since the reduced sensitivity to terrain variations is achieved at the cost of increased dispersions during the later part of the trajectory. Consequently, it is believed that some other scheme may have to be developed.

There is also the problem of LM landing at science sites and the associated rough approach terrain, which will probably dictate a different descent trajectory from the one used on early Apollo missions.

Thus, it is likely that targeting of new trajectories will be performed more than once in the future. While automatic targeting of the braking phase has been accomplished(1) the design of the visibility phase remained a trail and error procedure.

This work does not propose an automated method of optimizing a visibility phase targeting. It does, however, provide a method and a rationale for the design of a visibility phase conforming with the many constraints. Using the procedure outlined below it is possible to construct within a few hours a visibility phase which would be compatible with a braking phase and allow for good landing radar data acquisition.

In addition, a method for "extending" targets is proposed. The portion of the braking phase after throttle down has occurred, is continued for t seconds, providing targets for a new trajectory which lasts approximately t seconds longer but which passes at $t_{go} = t$ through approximately the same state vector which was the end of the old braking phase. Thus, the switching of the targets could occur at $t_{go} = t$, reducing the sensitivity to rough terrain.

Together with G. L. Bush's[1] and O. R. Pardo's work, the methodical targeting of a whole descent phase (with or without a "false" high gate) becomes possible. The procedure is to target the visibility phase first. Then, to use the automated procedure proposed by Bush and Pardo to target a braking phase ending where the visibility phase begins. If extended targeting (false high gate) is desired, the braking phase trajectory obtained above can be used to obtain the new targets.

2.0 DESIGN OF A VISIBILITY PHASE

2.1 Quadratic Guidance (See Appendix A for signs, coordinates, and time conventions)

In quadratic guidance, the acceleration is a second order polynomial of the quantity t_{go}, which is the time left until the end of the phase. Thus the acceleration (in each coordinate) can be written as

$$A = A_D + C_2' t_{go} + C_3' t_{go}^2 \tag{1}$$

(We shall abbreviate by writing t for t_{go}.) See Appendix B for nomenclature.

The velocity, which is the time integral of the acceleration can be written as a cubic function of time

$$V = V_D + A_D t + \frac{C_2'}{2} t^2 + \frac{C_3'}{3} t^3 = V_D + A_D t + C_2 t^2 + C_3 t^3 \tag{2}$$

and the distance ΔP from the end point of the phase (in any coordinate)

$$\Delta P = V_D t + A_D t^2 + \frac{C_2 t^3}{3} + \frac{C_3 t^4}{4} \tag{3}$$

2.2 The Constraints

The most restrictive constraint during the visibility phase is the maximum rate of descent as a function of altitude. For lack of a rigorous definition of this constraint we are forced to derive it by implication, from the restriction on the horizontal velocity as a function of the range and the flight path angle (page 5).

If we consider a trajectory passing through the point V_Z = 88 fps, Z = 2,000 ft (horizontal velocity vs. range constraint), and V_X = 88 fps x tan 16° = 25.3 fps and $X = X_D + 2{,}000 \tan 16° = 570 + X_D$ ft (16° FPA constraint), we obtain one point on the Altitude vs. Altitude rate constraint. The other point is the final conditions of the visibility phase, i.e., X_D = 60 ft, V_X = 3 fps. As a simplification we draw a line between the two points on the $X;\dot{X}$ phase plane and obtain Figure 1. The slope of the line is 25.3.

To be in the region which does not violate the constraint $\Delta X \geq 25.3\ \Delta V_X$. But

$$X - X_D = \Delta X = V_{DX}t + \frac{A_{DX}}{2} t^2 + \frac{C_2 t^3}{3} + \frac{C_3 t^4}{4}$$

$$V_X - V_{DX} = \Delta V_X = A_{DX}t + C_2 t^2 + C_3 t^3$$

Writing the inequality we obtain

$$C_2 \leq \frac{V_{DX}}{t(25.3 - \frac{t}{3})} - \frac{A_{DX}(25.3 - \frac{t}{2})}{t(25.3 - \frac{t}{3})} - \frac{C_3 t(25.3 - \frac{t}{4})}{(25.3 - \frac{t}{3})}$$

Since an analytical solution is difficult, we write values for different t_{go} and from Table A.

TABLE A

$(V_{XD} = 3 \text{ fps } X_D = 60 \text{ ft})$

t_{go} =	$C_2 \leq$ than
10	$.00178 - .0926\ A_{DX} - 10.37\ C_3$
20	$.00783 - .0413\ A_{DX} - 21.47\ C_3$
30	$.00633 - .02278\ A_{DX} - 34.75\ C_3$
40	$.00602 - .01163\ A_{DX} - 50.7\ C_3$
50	$.00657 - .00175\ A_{DX} - 72.8\ C_3$
60	$.00862 + .012\ A_{DX} - 117.7\ C_3$
70	$.01737 + .0523\ A_{DX} - 235.5\ C_3$

From previous experience with descent trajectories, we know that at the beginning of the visibility phase thrust acceleration cannot exceed approximately 11 fps^2 or "pulse up" of the descent propulsion system may occur. Since the lunar gravitation is 5.32 fps^2, and the inertial acceleration must be directed nearly along the flight path, the maximum acceleration during the visibility phase may not exceed 2.0 fps^2 in the X direction and 8.0 fps^2 in the Z direction for a flight path angle (FPA) = 16°.

The FPA must remain between 12° and 20° for the duration of the phase and be at 16° $\pm$ 1° when the range (Z) is 20,000 ft. In addition, at 2,000 ft range, t_{go} must be larger than 55 seconds and V_Z smaller than 88 fps. (The rationale for choosing 88 fps, which is exactly 60 mph, is not very clear and suggests that a few fps violation of the "speed limit" will not necessarily result in a ticket.)

Other constraints are imposed but they are satisfied almost automatically if all the above constraints are met.

To insure that the FPA remains at exactly 16° throughout the visibility phase, the following conditions must be satisfied.

$$\frac{A_{DX}}{A_{DZ}} = \frac{C_{2X}}{C_{2Z}} = \frac{C_{3X}}{C_{3Z}} = \frac{V_{DX}}{V_{DZ}} = \tan 16° \qquad (7)$$

These conditions cannot all be satisfied since we already know that $V_{DZ} = 0$ but V_{DX} is not, and that $A_{DX} = 0$ but A_{DZ} is not necessarily equal to zero. Thus, deviations from the nominal 16° FPA may occur during the last part of the visibility phase.

2.3 Some Design Tools

The design of a visibility phase must begin by choosing C_{2X} and C_{3X} that do not violate the descent rate constraint. When such constants are found, we may attempt to satisfy other conditions such as keeping the initial vertical acceleration within the bounds prescribed in the previous section, or increasing or decreasing the velocity or altitude at any point in the trajectory. This iteration process involves variations in C_2, C_3. We now attempt to define the influence of varying C_2 and C_3 on the trajectory parameters.

The visibility phase trajectory must pass through a nominal point. The nominal point (20,000 ft range) is the last point where a 3,000 ft redesignation is possible. Visibility is needed for M seconds prior to passing through that point to allow for seeing the obstacle and for deciding about the redesignation. The equations developed below are valid only for such a trajectory.

$$dV_I = \frac{\partial V_I}{\partial C_3} dC_3 + \frac{\partial V_I}{\partial t_I} \frac{dt_I}{dC_3} dC_3 \qquad (8)$$

The nominal point through which the LM must pass M seconds after the beginning of the visibility phase is at altitude 5,740 ft (20,000 ft tan 16°).

$$5{,}740 = V_D t_N + \frac{C_2 t_N^{\ 3}}{3} + \frac{C_3 t_N^{\ 4}}{4} \tag{9}$$

where the subscript N is associated with quantities at the nominal point.

Thus, $\frac{dt_N}{dC_3}$ can be found by differentiating (9) implicitly

$$0 = V_D dt_N + A_D t_N dt_N + C_2 t_N^{\ 2} dt_N + C_3 t_N^{\ 3} dt_N + \frac{t_N^{\ 4}}{4} dC_3$$

Grouping coefficients of dt_N we obtain;

$$(V_D + A_D t_N + C_2 t_N^{\ 2} + C_3 t_N^{\ 3})\ dt_N = V_N dt_N$$

$$\frac{dt_N}{dC_3} = \frac{t_N^{\ 4}}{4V_N} \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \tag{10}$$

Since the phase begins in any case for exactly M seconds before passing through the nominal point

$$\frac{dt_N}{dC_3} = \frac{dt_I}{dC_3} \quad \text{since } t_I - t_N = M = \text{const.}$$

Also,

$$\frac{\partial V_I}{\partial C_3} = t_I^{\ 3}$$

$$\frac{\partial V_I}{\partial t_I} = A_I.$$

Thus (8) may be written as

$$dV_I = \left(t_I^{\ 3} - \frac{t_N^{\ 4} A_I}{4V_N}\right) dC_3 \tag{11}$$

where: dV_I is the change in initial phase velocity

t_I is the duration of the phase on the previous iteration

t_N is the time from passing through the nominal altitude to end of phase on previous iteration

V_N is the velocity when passing through the nominal altitude on previous iteration

A_I is the acceleration at phase initiation on the previous iteration

dC_3 is the change in C_3.

By exactly the same procedure and the same notation we obtain:

$$dV_I = \left(t_I^2 - \frac{t_N^3}{3V_N} A_I \right) dC_2 \tag{12}$$

$$dX_I = \left(\frac{t_I^3}{3} - \frac{t_N^3}{3V_N} V_I \right) dC_2 \tag{13}$$

$$dX_I = \left(\frac{t_I^4}{4} - \frac{t_N^4}{4V_N} V_I \right) dC_3 \tag{14}$$

Precisely the same formulation holds for the Z direction. Even though the expression for V_N is different, it is easily verified that the expressions (11) - (14) are true also for the range component.

Thus, given a desired change in the initial position or velocity it is possible to find the necessary change in C_2 and C_3 to accomplish that change.

2.4 Some General Observations and Design Rules

1. The exact choice of C_2 and C_3 in the X direction is an intuitive process. Reasonable values may be picked out of Table A and checked for compliance with the Altitude vs. Altitude rate constraint. In general, it is preferable to increase C_2 rather than C_3 to minimize the initial acceleration. C_2 is increased until violation of the descent rate constraint occurs or the initial phase acceleration limit is exceeded. C_3 should be larger than zero only if violation of the former occurs first.

2. Changing C_2 influences the end of the phase as well as its beginning. Changing C_3 influences primarily the beginning of the phase (large t_{go}).

3. The targeting of the trajectory in the Z direction (downrange) can start after the targeting of the X direction is completed. We can start by letting $V_{DZ} = 0$; $\frac{C_{2X}}{C_{2Z}} = \frac{C_{3X}}{C_{3Z}} = \tan 16°$.

4. We must define A_{DZ}. Since the ratio $\frac{C_{2X}}{C_{2Z}}$ is the dominant fraction in the trajectory it should remain unchanged. A_{DZ} is now increased until violation of either

   (a) the 2,000 ft range velocity and t_{go} constraint, or

   (b) the initial Z acceleration limit occurs.

Generally (a) is violated first. Increasing C_{3Z} can now cause the trajectory to be flatter in the beginning of the phase and the downrange initial velocity to increase. C_{3Z} attains its maximum allowable value when (b) is just violated.

The targeting of the visibility phase is now completed.

2.5 Example - Design of a Visibility Phase for a "False High Gate" Trajectory

We first define the maximum allowable C_{2X}.

Using trial and error and the Altitude vs. Altitude rate constraint curve we find that C_2 can be increased to .005 for V_D = 3.0 fps and X_D = 60 ft. With C_2 = .005 and C_3 = 0 the LM passes through the nominal point at t_{go} = 144.3 seconds.

Thus for M = 50 seconds the phase duration is 194.3 seconds. The initial acceleration in the X direction is 1.943 fps^2, very near to the limit of 1.96 fps^2. Thus we keep C_{3X} = 0.

For the first iteration for the values of C_{2Z}, C_{3Z} we choose $C_{2Z} = C_{2X}/\tan 16° = .01744$

$C_{3Z} = 0.$

The maximum allowable A_{DZ} is approximately .4 fps^2 or the maximum velocity for 2,000 ft range will be exceeded. However, the velocity at the beginning of the phase is quite small and could be improved by introducing some positive valued C_{3Z}.

After a few iterations the following constants are found to satisfy the constraints:

$$A_{DZ} = .25$$

$$C_{2Z} = .01744$$

$$C_{3Z} = .000009.$$

When these values are translated into the usual targeting parameters we obtain final conditions

T_{go} = 194.3 seconds $\quad$ V_{DX} = 4.5 fps

J_{DZ} = .03488 fps^3 $\quad$ V_{DZ} = 0

A_{DZ} = .25 fps^2 $\quad$ P_{DZ} = 0

A_{DX} = 0 $\quad$ P_{DX} = 90 ft

and initial conditions

V_{XI} = 187.2 fps

P_{XI} = 12,862 ft

V_{ZI} = 772.99 fps

P_{ZI} = 50,568 ft

A_{XI} = 1.943 fps^2

A_{ZI} = 8.046 fps^2

$$|A_I| = 8.046^2 + (1.943 + 5.32)^2 = 10.84 \text{ fps}^2$$

3.0 EXTENDING TARGETS OF A BRAKING PHASE

3.1 The Method

In conjunction with a "false high gate" visibility phase one must target also a matching braking phase. The existing program[1] can assist in the targeting only if the target point of the braking phase is also the point at which the switching to the new target (of the visibility phase) occurs.

However, in "false high gate" trajectory the LM switches its aim point (i.e., targeting parameters) at time T (the switchover point) prior to reaching the braking phase aim point. Thus, we must find braking phase targets which would cause the LM to pass through the "switchover" point with the intended velocity and acceleration at t_{go} = T. Such targets can be defined by following the procedure outlined below.

We first employ the existing program to obtain target parameters for the switchover point. That is, we assume that the switchover point is the real end point of the braking phase. After running a nominal trajectory we obtain the position and velocity at $t_{go} = 0$ and at any other two t_{go}'s after throttle down has occurred. We use these data and the desired accelerations (at $t_{go} = 0$) to define C_2 a C_3 in each coordinate. Since

$$\Delta V_1 = A_D\, t_{go_1} + C_2\, t_{go_1}{}^2 + C_3\, t_{go_1}{}^3 \tag{15}$$

$$\Delta V_2 = A_D\, t_{go_2} + C_2\, t_{go_2}{}^2 + C_3\, t_{go_2}{}^3 \tag{16}$$

where ΔV_1 and ΔV_2 are the changes in velocity between the two points above and the switchover point. Since ΔV; t_{go} and A_D and known C_2 and C_3 can be determined.

Once these two parameters have been determined, the approximate targets for the "extended" trajectory can be found. Let T be the time added to the braking phase.

$$J_D = J_T + ST$$

$$J_T = J_D - ST \; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \tag{17}$$

$$A_D = A_T + J_T T + S\,\frac{T^2}{2}$$

$$A_T = A_D - J_T T - S\,\frac{T^2}{2}$$

$$A_T = A_D - (J_D - ST)T - S\,\frac{T^2}{2}$$

$$A_T = A_D - J_D T + \frac{1}{2}\,ST^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(18)$$

$$V_D = V_T + A_T T + J_T\,\frac{T^2}{2} + S\,\frac{T^3}{6}$$

$$V_T = V_D - A_T T - J_T\,\frac{T^2}{2} - S\,\frac{T^3}{6}$$

$$V_T = V_D - (A_D - J_D T + \frac{1}{2}\,ST^2)T - J_D\,\frac{T^2}{2} + S\,\frac{T^3}{2} - S\,\frac{T^3}{6}$$

$$V_T = V_D - A_D T + J_D T^2\;(-\frac{1}{2} + \frac{1}{2})ST^3 - J_D\,\frac{T^2}{2} - S\,\frac{T^3}{6}$$

$$V_T = V_D - A_D T + J_D\,\frac{T^2}{2} - \frac{1}{6}\,ST^3 \ldots\ldots\ldots\ldots\ldots\ldots(19)$$

$$P_D = P_T + V_T T + A_T\,\frac{T^2}{2} + J_T\,\frac{T^3}{6} + S\,\frac{T^4}{24}$$

$$P_T = P_D - V_T T - A_T\,\frac{T^2}{2} - J_T\,\frac{T^3}{6} - S\,\frac{T^4}{24}$$

$$\begin{aligned} P_T = {} & P_D - V_D T + A_D T^2 - J_D\,\frac{T^3}{2} + \frac{1}{6}\,ST^4 \\ & - (+A_D - J_D T + \frac{S}{2}\,T^2)\,\frac{T^2}{2} \\ & - (+J_D - ST)\,\frac{T^3}{6} - S\,\frac{T^4}{24} \\ = {} & P_D - V_D T + A_D\,\frac{T^2}{2} - J_D\,\frac{T^3}{6} + ST^4\;(-\frac{1}{24} + \frac{1}{6} + \frac{1}{6} - \frac{1}{4}) \end{aligned}$$

$$P_T = P_D - V_D T + \frac{A_D T^2}{2} - \frac{J_D T^3}{6} + \frac{1}{24}\,ST^4 \ldots\ldots\ldots\ldots(20)$$

in terms of C_2 and C_3; $J_D = 2C_2$, $S = 6C_3$.

(17') $$J_T = 2C_2 - 6C_3T$$

(18') $$A_T = A_D - 2C_2T + 3C_3T^2$$

(19') $$V_T = V_D - A_DT + C_2T^2 - C_3T^3$$

(20') $$P_T = P_D - V_DT + \frac{A_DT^2}{2} - \frac{C_2T^3}{3} + \frac{C_3T^4}{4}$$

3.2 Examples: Extending Targets by 20 Seconds

Using the targeting scheme of Reference 1 we obtain at $t_{go} = 115.631$ seconds:

$P_X = .57246082 \times 10^7$ ft

$P_Z = -.12354670 \times 10^6$ ft

$P_Y = 0$

$V_X = -.10478710 \times 10^3$ fps

$V_Z = .13654184 \times 10^4$ fps

$V_Y = 0$

$A_X = .345796 \times 10^1$ fps^2

$A_Z = -.856124 \times 10^1$ fps^2

and at $t_{go} = 0$

$$X_D = .5707395 \times 10^7 \text{ ft}$$

$$Y_D = 0$$

$$Z_D = -24{,}000 \text{ ft}$$

$$V_{DX} = -150 \text{ fps}$$

$$V_{DY} = 0$$

$$V_{DZ} = 350 \text{ fps}$$

$$A_{DX} = +.401 \text{ fps}^2$$

$$A_{DY} = 0$$

$$A_{DZ} = -8.78 \text{ fps}^2$$

Using equations (15) and (16) we obtain

$$C_{2X} = -.001393 \qquad C_{3X} = -.00004719$$

$$C_{2Z} = +.0028968 \qquad C_{3Z} = -.00002492$$

Using equations (17), (18), (19), and (20) we obtain

$$A_{XT} = .40092 \text{ fps}^2$$

$$V_{XT} = -141.80 \text{ fps}$$

$$X_T = 5704477 \text{ ft}$$

$$J_{TZ} = +.008784 \text{ fps}^3$$

$$A_{TZ} = -8.63422 \text{ fps}^2$$

$$V_{TZ} = 175.76 \text{ fps}$$

$$Z_T = -18747.3 \text{ ft}$$

The targets computed by this method are a first iteration in a procedure which leads to the final targets. In the example above, using the computed targets, we found that slight deviations from the intended state vector at T = 20 occurred. These differences (summarized in Table B) were judged to result from the period when the actual thrust and the computed thrust are not the same, i.e., prior to throttle down. As seen in Table B, the discrepancy is in the vertical velocity. Adjusting the target vertical velocity from -141.8 to -147.8 led to an almost perfect agreement in the "switchover" high gate state vector.

A second test of this procedure was carried out. A different trajectory similar to the old two phase descent trajectory was extended by 50 seconds.

Using the following data

at $t_{go} = 0$	at $t_{go} = 110.667$
$X_D = 5{,}711{,}986.9$ ft	$X = 5{,}722{,}426.0$ ft
$Z_D = -33{,}077$ ft	$Z = -144{,}987.1$ ft
$V_X = -149.3$ fps	$V_X = -24.834$ fps
$V_Z = 561.3$ fps	$V_Z = +1462.44$ fps
$A_{XD} = -1.454 \text{ fps}^2$	$A_X = +3.588 \text{ fps}^2$
$J_{DZ} = -9.829 \times 10^{-3} \text{ fps}^2$	

we computed

$$C_{2X} = +.009297 \qquad C_{2Z} = -.009829$$

$$C_{3X} = -.0000645 \qquad C_{3Z} = -.000031421$$

from which the new targets were computed to be

$$J_{TZ} = -.0192553 \text{ fps}^3$$

$$A_{TZ} = 9.928 \text{ fps}^2 \qquad A_{TX} = -2.86745 \text{ fps}^2$$

$$V_{TZ} = 130.04 \text{ fps} \qquad V_{TX} = -263.3 \text{ fps}$$

$$Z_T = 15{,}642.1 \text{ ft} \qquad X_T = 5{,}701{,}716.0 \text{ ft}$$

Again, as shown in Table C, slight discrepancies resulted this time in both the vertical and downrange velocities. After a single iteration (V_{TX} changed to -266 fps; V_{TZ} to +150 fps) the errors were almost nulled (Table C).

4.0 Acknowledgment

The author is indebted to G. L. Bush who was first to recognize the need for this work and whose suggestions and criticism steered the author more than once away from errors and misconceptions.

Irv Silberstein

2015-IS-acm

I. Silberstein

Attachments
- References
- Figures 1 and 2
- Tables B and C
- Appendices A and B

REFERENCES

1. G. L. Bush and O. R. Pardo, "Automation of LM Descent Braking Phase Targeting", Bellcomm Memorandum for File, May 17, 1968.

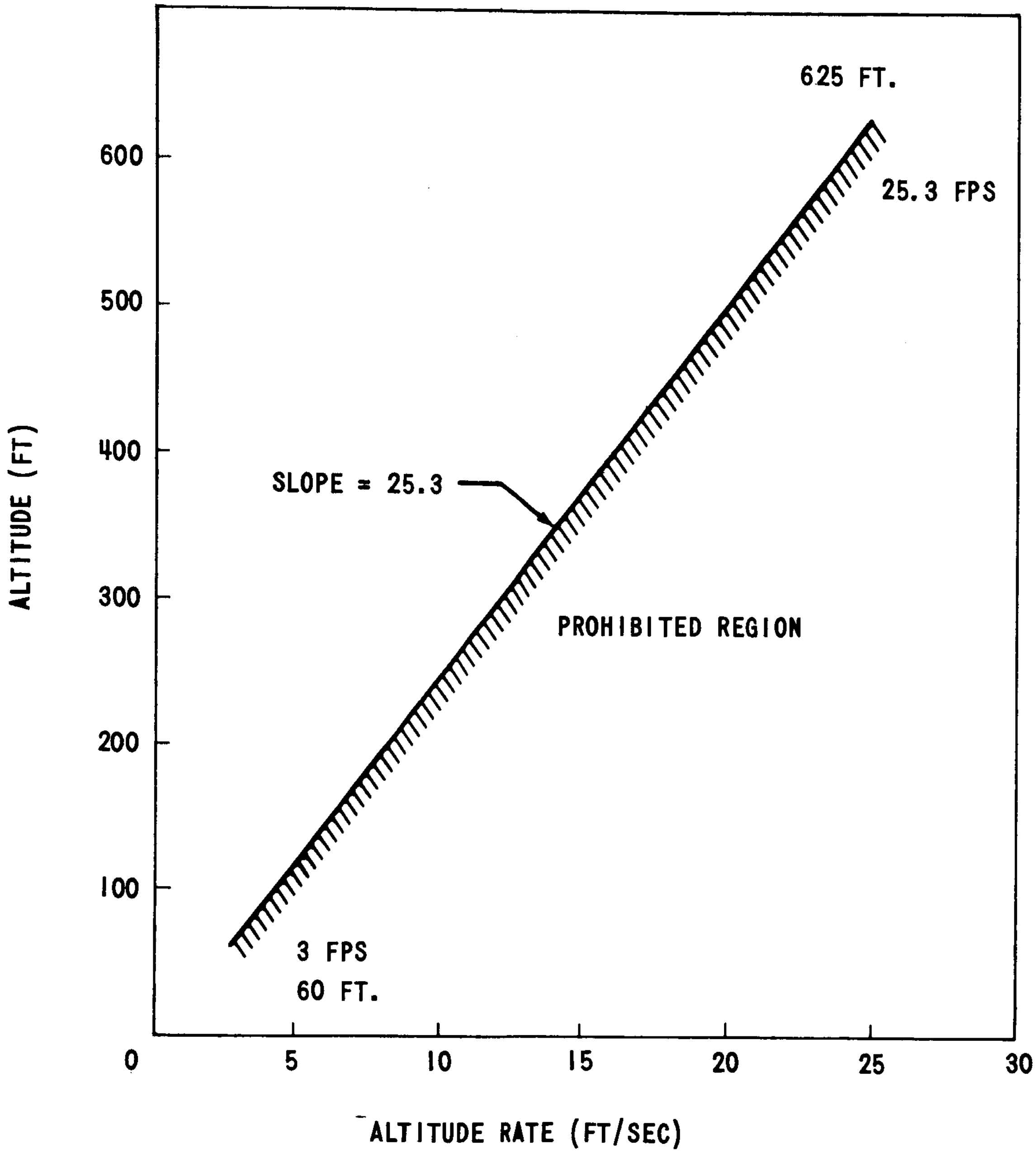

FIGURE I - ALTITUDE - ALTITUDE RATE CONSTRAINT

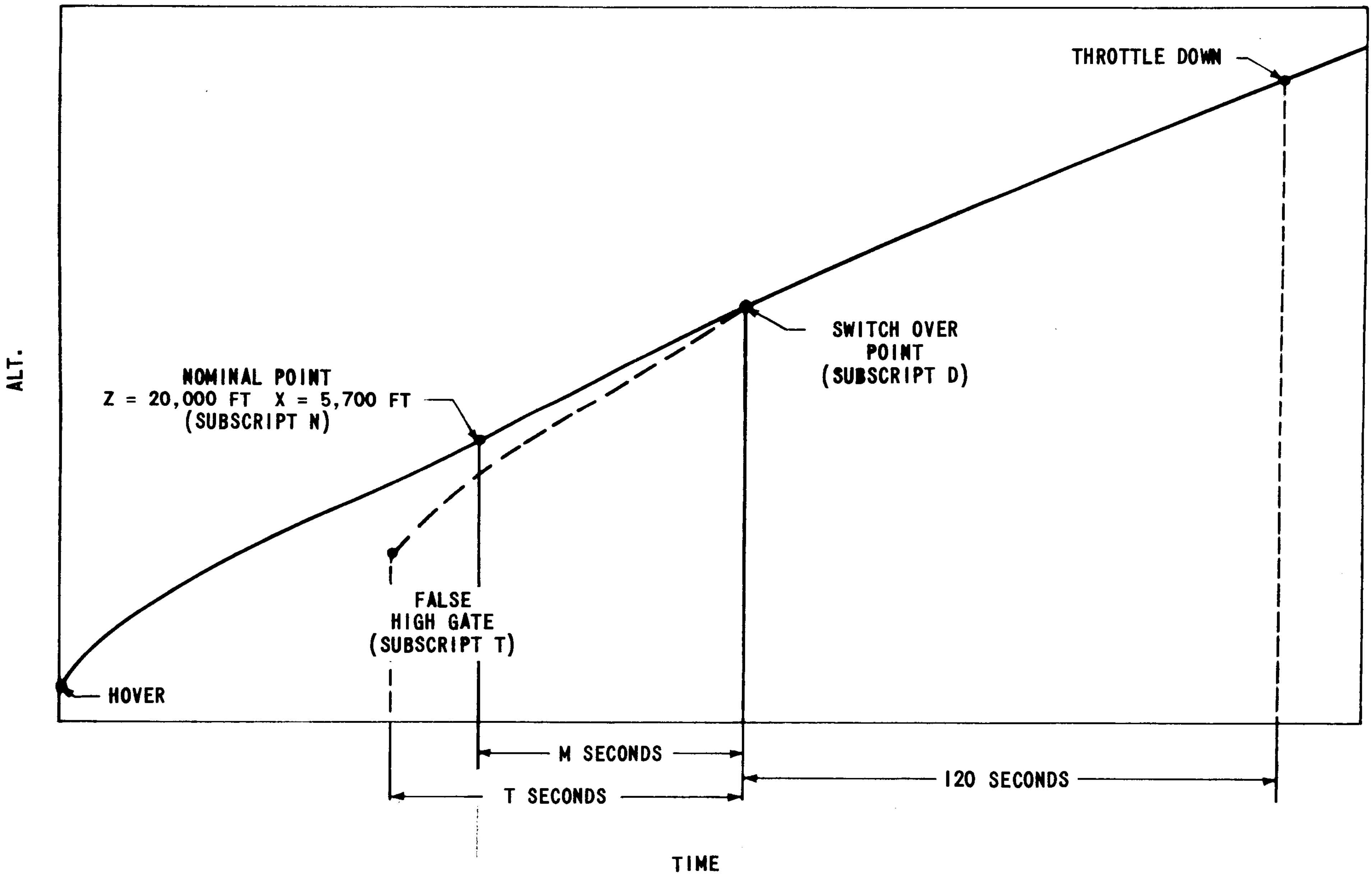

FIGURE 2 - MODIFIED TWO PHASE TRAJECTORY

TABLE B

STATE VECTORS

	INTENDED	FIRST TRIAL	FIRST CORRECTION
X	5714873.6	5711523.8	5715091.5
Z	50698	-36691.2	50515.7
V_X	-160.7	-155.7	-160.8
V_Z	772	789.0	769
TIME	469	489.9	469
THROTTLE TIME	119.6		137-20=117(sec)
ALTITUDE	12703	9246	12919

TABLE C

STATE VECTORS AT HIGH GATE

	INTENDED	FIRST TRIAL	FIRST CORRECTION
X	5711985.8	5711508.3	5711633.6
Z	-33077	-31778	-32719
V_X	-159.4	-156.8	-159.4
V_Z	561.2	541.8	561.5
TIME	500.6	501.9	499.9
THROTTLE T_{go}	118.6	163.3(-50=113.3)	161.6(-50=111.6)
ALTITUDE	9687	9201	9332

APPENDIX A

SIGNS, COORDINATES, AND TIME CONVENTIONS

The coordinate system used in this work is the usual guidance coordinate system with the sign of the Z coordinate changed. In this system the X coordinate is in the radial direction piercing the landing site continuously and the Z is in the uprange direction, orthogonal to the X. The time is considered positive in the sense of increasing $|T_{go}|$. That means that the LM is viewed as lifting off from hover through high gate and further towards PDI. However, we still refer to the "beginning" and "end" of a phase in the conventional meaning, i.e., in the order they occur in the actual flight.

APPENDIX B

NOMENCLATURE

X Y Z	A right hand triad as in Appendix A
$\overline{V}$	Velocity
$\overline{A}$	Acceleration
$\overline{J}$	Jerk $\frac{d\overline{A}}{dt}$
$\overline{S}$	Snap $\frac{d\overline{J}}{dt}$
T	Time of extension of the braking phase
M	Duration of flight from beginning of visibility phase to passage through the nominal point
C_{2X} C_{2Z}	Constants of quadratic guidance ($2J_{DX}$; $2J_{DZ}$)
C_{3X} C_{3Z}	Constants ($6S_X$; $6S_Z$)

Subscripts

D	Desired quantities at the end of a phase
T	Desired quantities at end of extended phase
N	Quantities at a nominal point which must lie in the trajectory
I	Initial conditions of any phase.

BELLCOMM, INC.

Subject: Targeting of the LM Descent Visibility Phase and "Extended" Braking Phase

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